Parametric Mortality Indexes: From Index Construction to Hedging Strategies

Chong It Tan, ASA, CERA
Nanyang Business School, Singapore

Joint work with Jackie Li, Johnny Siu-Hang Li and Uditha Balasooriya
Introduction

Construction of Mortality Indexes

Securitization

Hedging Strategies

Conclusion
Life Market

- the market for mortality/longevity-linked securities
- trading/hedging of longevity risk
- lack of liquidity
- creation of mortality indexes
Life Market

- the market for mortality/longevity-linked securities
- trading/hedging of longevity risk
- lack of liquidity
- creation of mortality indexes
Life Market

- the market for mortality/longevity-linked securities
- trading/hedging of longevity risk
- lack of liquidity
- creation of mortality indexes
Mortality Indexes

- model-free indexes
  - Credit Suisse Longevity Index (Credit Suisse 2005), LifeMetrics Index (J.P. Morgan 2007)
  - these indexes are either highly aggregate or specific
  - keep track of a large number of indexes

- model-based indexes
  - stochastic models
  - time-varying parameters
Mortality Indexes

- model-free indexes
  - Credit Suisse Longevity Index (Credit Suisse 2005), LifeMetrics Index (J.P. Morgan 2007)
  - these indexes are either highly aggregate or specific
  - keep track of a large number of indexes

- model-based indexes
  - stochastic models
  - time-varying parameters
Mortality Indexes

- model-free indexes
  - Credit Suisse Longevity Index (Credit Suisse 2005), LifeMetrics Index (J.P. Morgan 2007)
  - these indexes are either highly aggregate or specific
  - keep track of a large number of indexes

- model-based indexes
  - stochastic models
  - time-varying parameters
Model-based Mortality Indexes

- 3 primary criteria by Chan et al. (2014)
  - the new-data-invariant property: to ensure tractability
  - highly interpretable
  - varying age-pattern of mortality improvement
- six stochastic models in Dowd et al. (2010)
  - M1 (Lee-Carter model)
  - M2 (Renshaw-Haberman model)
  - M3 (Age-Period-Cohort model)
  - M5 (Cairns-Blake-Dowd model)
  - M6 (Cairns-Blake-Dowd model with cohort effects)
  - M7 (Cairns-Blake-Dowd model with cohort and quadratic age effects)
- Chan et al. (2014) found that M5 is the most suitable model
Model-based Mortality Indexes

- 3 primary criteria by Chan et al. (2014)
  - the new-data-invariant property: to ensure tractability
  - highly interpretable
  - varying age-pattern of mortality improvement

- six stochastic models in Dowd et al. (2010)
  - M1 (Lee-Carter model)
  - M2 (Renshaw-Haberman model)
  - M3 (Age-Period-Cohort model)
  - M5 (Cairns-Blake-Dowd model)
  - M6 (Cairns-Blake-Dowd model with cohort effects)
  - M7 (Cairns-Blake-Dowd model with cohort and quadratic age effects)

- Chan et al. (2014) found that M5 is the most suitable model
Model-based Mortality Indexes

- 3 primary criteria by Chan et al. (2014)
  - the new-data-invariant property: to ensure tractability
  - highly interpretable
  - varying age-pattern of mortality improvement

- six stochastic models in Dowd et al. (2010)
  - M1 (Lee-Carter model)
  - M2 (Renshaw-Haberman model)
  - M3 (Age-Period-Cohort model)
  - M5 (Cairns-Blake-Dowd model)
  - M6 (Cairns-Blake-Dowd model with cohort effects)
  - M7 (Cairns-Blake-Dowd model with cohort and quadratic age effects)

- Chan et al. (2014) found that M5 is the most suitable model
Model-based Mortality Indexes

- 3 primary criteria by Chan et al. (2014)
  - the new-data-invariant property: to ensure tractability
  - highly interpretable
  - varying age-pattern of mortality improvement
- six stochastic models in Dowd et al. (2010)
  - M1 (Lee-Carter model)
  - M2 (Renshaw-Haberman model)
  - M3 (Age-Period-Cohort model)
  - M5 (Cairns-Blake-Dowd model)
  - M6 (Cairns-Blake-Dowd model with cohort effects)
  - M7 (Cairns-Blake-Dowd model with cohort and quadratic age effects)
- Chan et al. (2014) found that M5 is the most suitable model
Adapting Mortality Models

- model with age-specific parameters (M1)
  - estimate the model parameters using a restricted sample period \([t_{\text{start}}, t_{\text{mid}}]\)
  - keep the age-specific parameters fixed when we update the model for sample period \([t_{\text{mid}}, t_{\text{end}}]\)
- model with cohort effect parameters (M6 and M7)
  - estimate the time-varying parameters first
  - estimate the cohort effect parameters from the residuals
Adapting Mortality Models

- model with age-specific parameters (M1)
  - estimate the model parameters using a restricted sample period \([t_{start}, t_{mid}]\)
  - keep the age-specific parameters fixed when we update the model for sample period \([t_{mid}, t_{end}]\)

- model with cohort effect parameters (M6 and M7)
  - estimate the time-varying parameters first
  - estimate the cohort effect parameters from the residuals
Adapting Mortality Models

- model with age-specific parameters (M1)
  - estimate the model parameters using a restricted sample period \([t_{start}, t_{mid}]\)
  - keep the age-specific parameters fixed when we update the model for sample period \([t_{mid}, t_{end}]\)
- model with cohort effect parameters (M6 and M7)
  - estimate the time-varying parameters first
  - estimate the cohort effect parameters from the residuals
Adapting Mortality Models

- model with both age-specific and cohort effect parameters (M2 and M3)
  - keep the age-specific parameters fixed
  - estimate the time-varying parameters first
  - estimate the cohort effect parameters from the residuals
Adapting Mortality Models

- **M1**: adapted M1
  \[
  \ln (m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)}
  \]

- **M2**: adapted M2
  \[
  \ln (m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)} + \beta_x^{(3)} \gamma_t^{(3)}
  \]

- **M3**: adapted M3
  \[
  \ln (m_{x,t}) = \beta_x^{(1)} + n_a^{-1} \kappa_t^{(2)} + n_a^{-1} \gamma_t^{(3)}
  \]

- **M6**: adapted M6
  \[
  \ln \left( \frac{q_{x,t}}{1-q_{x,t}} \right) = \kappa_t^{(1)} + \kappa_t^{(2)} (x - \bar{x}) + \gamma_t^{(3)}
  \]

- **M7**: adapted M7
  \[
  \ln \left( \frac{q_{x,t}}{1-q_{x,t}} \right) = \kappa_t^{(1)} + \kappa_t^{(2)} (x - \bar{x}) + \kappa_t^{(3)} ((x - \bar{x})^2 - \hat{\sigma}_x^2) + \gamma_t^{(4)}
  \]
Adapting Mortality Models

- **M1**: adapted M1
  \[ \ln(m_{x,t}) = \beta_x^{(1)*} + \beta_x^{(2)*} \kappa_t^{(2)} \]

- **M2**: adapted M2
  \[ \ln(m_{x,t}) = \beta_x^{(1)*} + \beta_x^{(2)*} \kappa_t^{(2)} + \beta_x^{(3)*} \gamma_{t-x}^{(3)*} \]

- **M3**: adapted M3
  \[ \ln(m_{x,t}) = \beta_x^{(1)*} + n_{a}^{-1} \kappa_t^{(2)} + n_{a}^{-1} \gamma_{t-x}^{(3)*} \]

- **M6**: adapted M6
  \[ \ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)} (x - \bar{x}) + \gamma_{t-x}^{(3)*} \]

- **M7**: adapted M7
  \[ \ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)} (x - \bar{x}) + \kappa_t^{(3)} ((x - \bar{x})^2 - \hat{\sigma}^2_{x}) + \gamma_{t-x}^{(4)*} \]
Adapting Mortality Models

- **M1**: adapted M1
  \[ \ln (m_{x,t}) = \beta^{(1)*}_x + \beta^{(2)*}_x \kappa^{(2)}_t \]

- **M2**: adapted M2
  \[ \ln (m_{x,t}) = \beta^{(1)*}_x + \beta^{(2)*}_x \kappa^{(2)}_t + \beta^{(3)*}_x \gamma^{(3)*}_{t-x} \]

- **M3**: adapted M3
  \[ \ln (m_{x,t}) = \beta^{(1)*}_x + n^{-1}_a \kappa^{(2)}_t + n^{-1}_a \gamma^{(3)*}_{t-x} \]

- **M6**: adapted M6
  \[ \ln \left( \frac{q_{x,t}}{1-q_{x,t}} \right) = \kappa^{(1)}_t + \kappa^{(2)}_t (x - \bar{x}) + \gamma^{(3)*}_{t-x} \]

- **M7**: adapted M7
  \[ \ln \left( \frac{q_{x,t}}{1-q_{x,t}} \right) = \kappa^{(1)}_t + \kappa^{(2)}_t (x - \bar{x}) + \kappa^{(3)}_t \left( (x - \bar{x})^2 - \hat{\sigma}^2_x \right) + \gamma^{(4)*}_{t-x} \]
Adapting Mortality Models

- **M1**: adapted M1
  \[ \ln (m_{x,t}) = \beta_x^{(1)*} + \beta_x^{(2)*} \kappa_t^{(2)} \]

- **M2**: adapted M2
  \[ \ln (m_{x,t}) = \beta_x^{(1)*} + \beta_x^{(2)*} \kappa_t^{(2)} + \beta_x^{(3)*} \gamma_{t-x}^{(3)*} \]

- **M3**: adapted M3
  \[ \ln (m_{x,t}) = \beta_x^{(1)*} + n^{-1} \kappa_t^{(2)} + n^{-1} \gamma_{t-x}^{(3)*} \]

- **M6**: adapted M6
  \[ \ln \left( \frac{q_{x,t}}{1-q_{x,t}} \right) = \kappa_t^{(1)} + \kappa_t^{(2)} (x - \bar{x}) + \gamma_{t-x}^{(3)*} \]

- **M7**: adapted M7
  \[ \ln \left( \frac{q_{x,t}}{1-q_{x,t}} \right) = \kappa_t^{(1)} + \kappa_t^{(2)} (x - \bar{x}) + \kappa_t^{(3)} ((x - \bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}^{(4)*} \]
Constructing Mortality Indexes

- gender-specific mortality data from 10 populations
  - Australasia: Australia (AUS), New Zealand (NZL)
  - East Asia: Taiwan (TWN), Japan (JPN)
  - Nordic region: Norway (NOR), Sweden (SWE)
  - Western Europe: England and Wales (EW), France (FRA)
  - North America: Canada (CAN), United States (USA)
- data source: Human Mortality Database
- age range: 40-90
- data sample period for NZL: [1950,1993], [1994,2008]
- data sample period for others: [1950,1994], [1995,2009]
Constructing Mortality Indexes

- gender-specific mortality data from 10 populations
  - Australasia: Australia (AUS), New Zealand (NZL)
  - East Asia: Taiwan (TWN), Japan (JPN)
  - Nordic region: Norway (NOR), Sweden (SWE)
  - Western Europe: England and Wales (EW), France (FRA)
  - North America: Canada (CAN), United States (USA)

- data source: Human Mortality Database
- age range: 40-90
- data sample period for NZL: [1950,1993], [1994,2008]
- data sample period for others: [1950,1994], [1995,2009]
Constructing Mortality Indexes

- gender-specific mortality data from 10 populations
  - Australasia: Australia (AUS), New Zealand (NZL)
  - East Asia: Taiwan (TWN), Japan (JPN)
  - Nordic region: Norway (NOR), Sweden (SWE)
  - Western Europe: England and Wales (EW), France (FRA)
  - North America: Canada (CAN), United States (USA)
- data source: Human Mortality Database
- age range: 40-90
- data sample period for NZL: [1950,1993], [1994,2008]
- data sample period for others: [1950,1994], [1995,2009]
Constructing Mortality Indexes

- maximum likelihood estimation (MLE)
- model selection criterion
  - reduction in log-likelihoods between original model and adapted model
  - Bayesian Information Criterion (BIC)
- M7* gives the best BIC values and the smallest reductions in log-likelihood values
- construct mortality indexes using $\kappa_t^{(1)}, \kappa_t^{(2)}, \kappa_t^{(3)}$ in M7*
Constructing Mortality Indexes

- maximum likelihood estimation (MLE)
- model selection criterion
  - reduction in log-likelihoods between original model and adapted model
  - Bayesian Information Criterion (BIC)
- M7* gives the best BIC values and the smallest reductions in log-likelihood values
- construct mortality indexes using \( \kappa_t^{(1)}, \kappa_t^{(2)}, \kappa_t^{(3)} \) in M7*
Constructing Mortality Indexes

| Pop | Males | | | | | Females | | | |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| AUS | 3734  | 3125  | 1624  | 335   | 19    | 1045  | 6249  | 1431  | 1150  | 24    |
| CAN | 6948  | 24836 | 1404  | 203   | 15    | 1060  | 6809  | 805   | 1313  | 194   |
| EW  | 13886 | 2362  | 2600  | 90    | 83    | 9566  | 1633  | 2353  | 1252  | 27    |
| FRA | 2051  | 2282  | 3466  | 4539  | 1820  | 7847  | 10668 | 4107  | 14868 | 545   |
| JPN | 24212 | 1805  | 4118  | 2655  | 537   | 88416 | 2707  | 3853  | 13185 | 261   |
| NZL | 972   | 830   | 399   | 32    | 1     | 555   | 538   | 389   | 191   | 38    |
| NOR | 359   | 2351  | 437   | 54    | 4     | 94    | 267   | 377   | 480   | 27    |
| SWE | 979   | 364   | 902   | 190   | 12    | 361   | 206   | 892   | 943   | 76    |
| TWN | 8033  | 6151  | 951   | 910   | 5     | 3415  | 1886  | 718   | 505   | 15    |
| USA | 53358 | 14229 | 10554 | 8860  | 1766  | 20218 | 11028 | 7202  | 14371 | 2224  |
## Constructing Mortality Indexes

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>15766</td>
<td>14937</td>
<td>11419</td>
<td>11942</td>
<td>8894</td>
<td>8199</td>
<td>10051</td>
<td>20938</td>
<td>10810</td>
<td>17545</td>
<td>10452</td>
<td>7965</td>
</tr>
<tr>
<td>CAN</td>
<td>22776</td>
<td>58868</td>
<td>11543</td>
<td>10697</td>
<td>9298</td>
<td>8781</td>
<td>10677</td>
<td>22564</td>
<td>10066</td>
<td>17673</td>
<td>11453</td>
<td>8752</td>
</tr>
<tr>
<td>EW</td>
<td>39538</td>
<td>14465</td>
<td>14497</td>
<td>16137</td>
<td>9887</td>
<td>9454</td>
<td>30549</td>
<td>12765</td>
<td>13763</td>
<td>24662</td>
<td>12298</td>
<td>9097</td>
</tr>
<tr>
<td>FRA</td>
<td>15899</td>
<td>14421</td>
<td>16359</td>
<td>56935</td>
<td>23612</td>
<td>13372</td>
<td>26739</td>
<td>30858</td>
<td>17351</td>
<td>121168</td>
<td>41037</td>
<td>11041</td>
</tr>
<tr>
<td>JPN</td>
<td>62618</td>
<td>14144</td>
<td>19169</td>
<td>40456</td>
<td>19438</td>
<td>12627</td>
<td>187997</td>
<td>15358</td>
<td>17880</td>
<td>132618</td>
<td>38947</td>
<td>12164</td>
</tr>
<tr>
<td>NZL</td>
<td>8852</td>
<td>9100</td>
<td>7752</td>
<td>6833</td>
<td>6902</td>
<td>6848</td>
<td>7880</td>
<td>8375</td>
<td>7605</td>
<td>7869</td>
<td>7213</td>
<td>6801</td>
</tr>
<tr>
<td>NOR</td>
<td>7801</td>
<td>12354</td>
<td>7989</td>
<td>7338</td>
<td>7057</td>
<td>7021</td>
<td>7061</td>
<td>7922</td>
<td>7621</td>
<td>9977</td>
<td>7716</td>
<td>6849</td>
</tr>
<tr>
<td>SWE</td>
<td>9668</td>
<td>8935</td>
<td>9499</td>
<td>9175</td>
<td>8035</td>
<td>7652</td>
<td>8269</td>
<td>8460</td>
<td>9273</td>
<td>14262</td>
<td>9405</td>
<td>7589</td>
</tr>
<tr>
<td>TWN</td>
<td>24720</td>
<td>21321</td>
<td>10446</td>
<td>15448</td>
<td>10333</td>
<td>8580</td>
<td>14829</td>
<td>12271</td>
<td>9449</td>
<td>12044</td>
<td>8820</td>
<td>7888</td>
</tr>
<tr>
<td>USA</td>
<td>123233</td>
<td>40388</td>
<td>34770</td>
<td>102543</td>
<td>40562</td>
<td>17818</td>
<td>54680</td>
<td>33198</td>
<td>25935</td>
<td>122063</td>
<td>52332</td>
<td>16553</td>
</tr>
</tbody>
</table>
K-forward

- standardized mortality-linked security
- a swap between a fixed amount (pre-determined forward value) and a random amount (realized index value) related to one of the three indexes in a reference year
- K1-forward, K2-forward, K3-forward

\[
Y \times (\tilde{\kappa}^{(i)}_t - \kappa^{(i)}_t), \quad i = 1, 2, 3.
\]
K-forward

- standardized mortality-linked security
- a swap between a fixed amount (pre-determined forward value) and a random amount (realized index value) related to one of the three indexes in a reference year
- K1-forward, K2-forward, K3-forward

\[ Y \times (\tilde{\kappa}_t^{(i)} - \kappa_t^{(i)}), \quad i = 1, 2, 3. \]
K-forward

- standardized mortality-linked security
- a swap between a fixed amount (pre-determined forward value) and a random amount (realized index value) related to one of the three indexes in a reference year
- K1-forward, K2-forward, K3-forward

\[ Y \times (\tilde{\kappa}_{t*}^{(i)} - \kappa_{t*}^{(i)}), \quad i = 1, 2, 3. \]
Key K-duration (KKD)

- similar to key q-duration (Li and Luo, 2012) for q-forwards
- ‘key’: K-forwards are only available in certain key years $t_1, t_2, \ldots, t_n$
- measures the change in the value of a liability with respect to a small change in a key K-index
- two assumptions
  - a shock in $\kappa_{t,j}^{(i)}$ is accompanied by a level shift in $\kappa_t^{(i)}$ over the period of $t_j \leq t < t_{j+1}$
  - the shock on $\kappa_t^{(i)}$ has no impact on $\kappa_t^{(h)}$ for all $i \neq h$ and $t$
- $KKD_i(P(\kappa), j) = \frac{\partial P(\kappa)}{\partial \kappa_{t,j}^{(i)}}$
Key K-duration (KKD)

- similar to key q-duration (Li and Luo, 2012) for q-forwards
- ‘key’: K-forwards are only available in certain key years $t_1, t_2, \ldots, t_n$
- measures the change in the value of a liability with respect to a small change in a key K-index
- two assumptions
  - a shock in $\kappa_{t,j}^{(i)}$ is accompanied by a level shift in $\kappa_{t}^{(i)}$ over the period of $t_j \leq t < t_{j+1}$
  - the shock on $\kappa_{t}^{(i)}$ has no impact on $\kappa_{t}^{(h)}$ for all $i \neq h$ and $t$
- $KKD_i(P(\kappa), j) = \frac{\partial P(\kappa)}{\partial \kappa_{t,j}^{(i)}}$
Key K-duration (KKD)

- similar to key q-duration (Li and Luo, 2012) for q-forwards
- ‘key’: K-forwards are only available in certain key years $t_1, t_2, \ldots, t_n$
- measures the change in the value of a liability with respect to a small change in a key K-index
- two assumptions
  - a shock in $\kappa_{t,j}^{(i)}$ is accompanied by a level shift in $\kappa_t^{(i)}$ over the period of $t_j \leq t < t_{j+1}$
  - the shock on $\kappa_t^{(i)}$ has no impact on $\kappa_t^{(h)}$ for all $i \neq h$ and $t$
- $KKD_i(P(\kappa), j) = \frac{\partial P(\kappa)}{\partial \kappa_{t,j}^{(i)}}$
Building Longevity Hedge

- KKD strategy
- KKD of liability portfolio is estimated numerically
- KKD of K-forward can be derived analytically
  - KKD of liability portfolio = KKD of hedge portfolio consisting of K-forwards, for each key K-index in each key year
  - determine the required notional amounts of K1-forward, K2-forward and K3-forward separately in respective key years
Building Longevity Hedge

- KKD strategy
- KKD of liability portfolio is estimated numerically
- KKD of K-forward can be derived analytically
- KKD of liability portfolio = KKD of hedge portfolio consisting of K-forwards, for each key K-index in each key year
- determine the required notional amounts of K1-forward, K2-forward and K3-forward separately in respective key years
Hedging Illustrations: Single Cohort

- pension plan coverage: $1 at the beginning of each year from age 65 until the pensioner dies or attains age 91
- mortality data: English and Welsh males, ages 40-90, [1950,2009]
- reference years: 2015, 2020, 2025, 2030
- interest rate: 3% flat
- parametric bootstrap (see Brouhns et al., 2005) simulation: 5000 scenarios
- amount of longevity risk reduction:
  \[ R = 1 - \frac{\text{variance of PV of unexpected cash flows after hedging}}{\text{variance of PV of unexpected cash flows without hedging}} \]
- simulation models: M7*, M5, M3, M2, MRW (Bell, 1997)
Hedging Illustrations: Single Cohort

- pension plan coverage: $1 at the beginning of each year from age 65 until the pensioner dies or attains age 91
- mortality data: English and Welsh males, ages 40-90, [1950,2009]
- reference years: 2015, 2020, 2025, 2030
- interest rate: 3% flat
- parametric bootstrap (see Brouhns et al., 2005) simulation: 5000 scenarios
- amount of longevity risk reduction:
  \[ R = 1 - \frac{\text{variance of PV of unexpected cash flows after hedging}}{\text{variance of PV of unexpected cash flows without hedging}} \]
- simulation models: M7*, M5, M3, M2, MRW (Bell, 1997)
Hedging Illustrations: Single Cohort

- pension plan coverage: $1 at the beginning of each year from age 65 until the pensioner dies or attains age 91
- mortality data: English and Welsh males, ages 40-90, [1950,2009]
- reference years: 2015, 2020, 2025, 2030
- interest rate: 3% flat
- parametric bootstrap (see Brouhns et al., 2005) simulation: 5000 scenarios
- amount of longevity risk reduction:
  \[ R = 1 - \frac{\text{variance of PV of unexpected cash flows after hedging}}{\text{variance of PV of unexpected cash flows without hedging}} \]
- simulation models: M7*, M5, M3, M2, MRW (Bell, 1997)
Hedging Illustrations: Single Cohort

- Pension plan coverage: $1 at the beginning of each year from age 65 until the pensioner dies or attains age 91
- Mortality data: English and Welsh males, ages 40-90, [1950,2009]
- Reference years: 2015, 2020, 2025, 2030
- Interest rate: 3% flat
- Parametric bootstrap (see Brouhns et al., 2005) simulation: 5000 scenarios
- Amount of longevity risk reduction:
  \[ R = 1 - \frac{\text{variance of PV of unexpected cash flows after hedging}}{\text{variance of PV of unexpected cash flows without hedging}} \]
- Simulation models: M7*, M5, M3, M2, MRW (Bell, 1997)
Hedging Illustrations: Single Cohort

- KKD strategy: simple calibration
- Optimal hedge: simulations + numerical optimization

<table>
<thead>
<tr>
<th>Simulation model</th>
<th>KKD strategy</th>
<th>Optimal hedge</th>
</tr>
</thead>
<tbody>
<tr>
<td>M7*</td>
<td>94.7%</td>
<td>97.3%</td>
</tr>
<tr>
<td>M5</td>
<td>96.0%</td>
<td>99.1%</td>
</tr>
<tr>
<td>M3</td>
<td>95.6%</td>
<td>96.6%</td>
</tr>
<tr>
<td>M2</td>
<td>95.2%</td>
<td>95.8%</td>
</tr>
<tr>
<td>MRW</td>
<td>93.5%</td>
<td>94.0%</td>
</tr>
</tbody>
</table>
Hedging Illustrations: Single Cohort

Model M7*

- Unhedged
- K1 only
- K2 only
- K3 only
- K1 & K2 & K3

Present value of unexpected cash flow density
Hedging Illustrations: Single Cohort

- Sampling risk (small-sample risk): smaller $R$ for smaller number of pensioners
- Sensitivity tests
  - interest rate: $R$ is not sensitive to the interest rate assumption
  - availability of K-forwards: more key years and/or smaller separation between two adjacent key years produce more effective hedge
  - age range: $R$ of K1- and K3-forward increase for an older age range, but $R$ of K2-forward drops
- advanced ages: satisfactory $R$ for pension coverage until age 101
Hedging Illustrations: Single Cohort

- Sampling risk (small-sample risk): smaller $R$ for smaller number of pensioners
- Sensitivity tests
  - interest rate: $R$ is not sensitive to the interest rate assumption
  - availability of K-forwards: more key years and/or smaller separation between two adjacent key years produce more effective hedge
  - age range: $R$ of K1- and K3-forward increase for an older age range, but $R$ of K2-forward drops
- advanced ages: satisfactory $R$ for pension coverage until age 101
Hedging Illustrations: Single Cohort

- Sampling risk (small-sample risk): smaller $R$ for smaller number of pensioners
- Sensitivity tests
  - interest rate: $R$ is not sensitive to the interest rate assumption
  - availability of K-forwards: more key years and/or smaller separation between two adjacent key years produce more effective hedge
    - age range: $R$ of K1- and K3-forward increase for an older age range, but $R$ of K2-forward drops
- advanced ages: satisfactory $R$ for pension coverage until age 101
Hedging Illustrations: Single Cohort

- Sampling risk (small-sample risk): smaller $R$ for smaller number of pensioners
- Sensitivity tests
  - interest rate: $R$ is not sensitive to the interest rate assumption
  - availability of K-forwards: more key years and/or smaller separation between two adjacent key years produce more effective hedge
  - age range: $R$ of K1- and K3-forward increase for an older age range, but $R$ of K2-forward drops
- advanced ages: satisfactory $R$ for pension coverage until age 101
Hedging Illustrations: Single Cohort

- Sampling risk (small-sample risk): smaller $R$ for smaller number of pensioners
- Sensitivity tests
  - interest rate: $R$ is not sensitive to the interest rate assumption
  - availability of K-forwards: more key years and/or smaller separation between two adjacent key years produce more effective hedge
  - age range: $R$ of K1- and K3-forward increase for an older age range, but $R$ of K2-forward drops
- advanced ages: satisfactory $R$ for pension coverage until age 101
Hedging Illustrations: Multiple Cohorts

- consider a multi-cohort pension plan with a coverage from age 60 to 91
- with both active members (ages 50-59) and retirement pensioners (ages 60-90)
- compare K-forward hedge with q-forward hedge
- K-forward: reference year
- q-forward: reference age and reference year
- K-forward hedge is easier to calibrate using key K-index
- q-forward hedge requires key cohorts and key q-rates
Hedging Illustrations: Multiple Cohorts

- consider a multi-cohort pension plan with a coverage from age 60 to 91
- with both active members (ages 50-59) and retirement pensioners (ages 60-90)
- compare K-forward hedge with q-forward hedge
- K-forward: reference year
- q-forward: reference age and reference year
- K-forward hedge is easier to calibrate using key K-index
- q-forward hedge requires key cohorts and key q-rates
Hedging Illustrations: Multiple Cohorts

- consider a multi-cohort pension plan with a coverage from age 60 to 91
- with both active members (ages 50-59) and retirement pensioners (ages 60-90)
- compare K-forward hedge with q-forward hedge
- K-forward: reference year
- q-forward: reference age and reference year
- K-forward hedge is easier to calibrate using key K-index
- q-forward hedge requires key cohorts and key q-rates
Hedging Illustrations: Multiple Cohort

**Pensioners**

- **Unhedged**
- **15 K–forwards**
- **15 q–forwards**

**Plan members**

- **Unhedged**
- **15 K–forwards**
- **24 q–forwards**

Present value of unexpected cash flow density
Hedging Illustrations: Multiple Cohorts

- the number of instruments required to produce a satisfactory hedge using K-forward remains the same, but that of q-forward rises with a larger number of cohorts
- due to the reference rates of K-forward and q-forward contracts
- K-forwards are potentially more liquid
Hedging Illustrations: Multiple Cohorts

- the number of instruments required to produce a satisfactory hedge using K-forward remains the same, but that of q-forward rises with a larger number of cohorts
- due to the reference rates of K-forward and q-forward contracts
- K-forwards are potentially more liquid
Concluding Remarks

- adapting mortality models to achieve the new-data-invariant property
- constructing mortality indexes using model M7*
- securitization of K-forward
- KKD hedging strategy yields excellent longevity risk reduction
- K-forwards are potentially more liquid for hedging purpose
Concluding Remarks

- adapting mortality models to achieve the new-data-invariant property
- constructing mortality indexes using model M7*
- securitization of K-forward
- KKD hedging strategy yields excellent longevity risk reduction
- K-forwards are potentially more liquid for hedging purpose
Concluding Remarks

- adapting mortality models to achieve the new-data-invariant property
- constructing mortality indexes using model M7*
- securitization of K-forward
- KKD hedging strategy yields excellent longevity risk reduction
- K-forwards are potentially more liquid for hedging purpose
Future Research

- convexity measure in calibrating the hedge
- dynamic hedging
- population basis risk
Thank you!